Basic Terms

Unit 1: Lesson 11

Transformations in the Coordinate Plane

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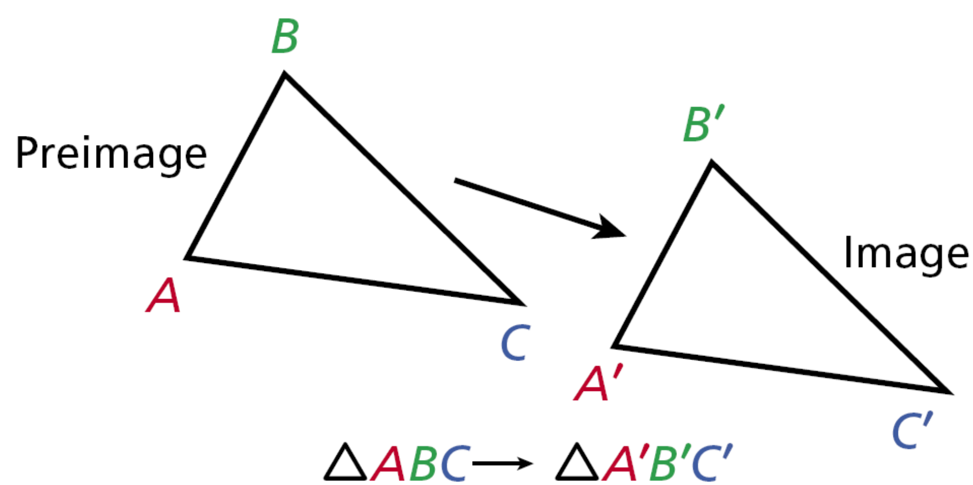
Objectives and Student Expectations

- TEKS: G2B, G10A
- The student will make conjectures about angles, lines and polygons using a variety of approaches including transformations.
- The student will use congruence transformations to make conjectures and justify properties of figures.

Basic Terms

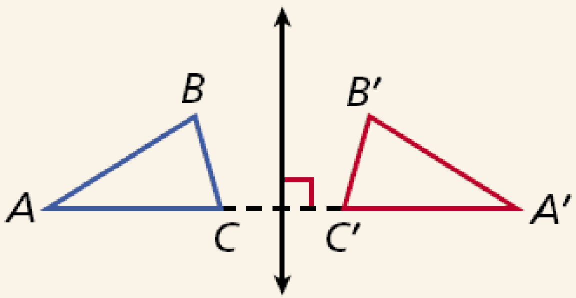
A **transformation** is a change in the position, size, or shape of a figure. The original figure is called the **preimage**. The resulting figure is called the **image**. A transformation *maps* the preimage to the image. Arrow notation (→) is used to describe a transformation, and primes (′) are used to label the image.



Basic Terms

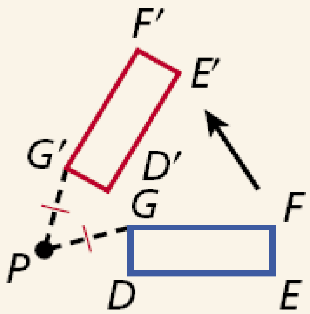
Transformations

REFLECTION



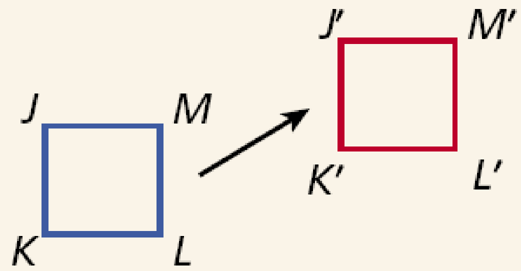
A **reflection** (or *flip*) is a transformation across a line, called the line of reflection. Each point and its image are the same distance from the line of reflection.

ROTATION



A **rotation** (or *turn*) is a transformation about a point *P*, called the center of rotation. Each point and its image are the same distance from *P*.

TRANSLATION

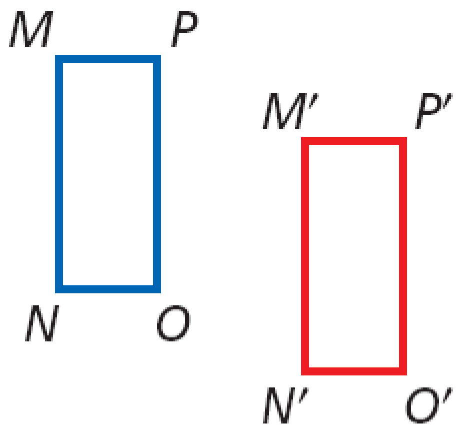


A **translation** (or *slide*) is a transformation in which all the points of a figure move the same distance in the same direction.

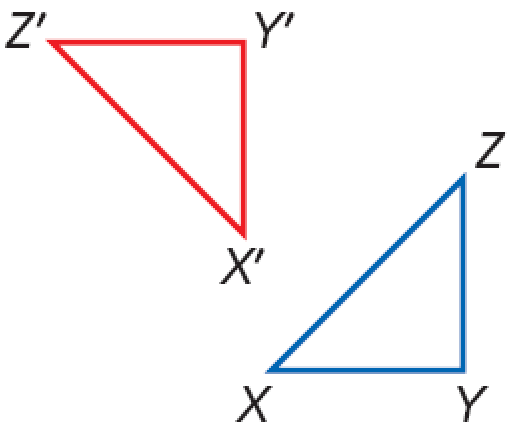
Example: 1

Identify each transformation. Then use arrow notation to describe the transformation.

a.



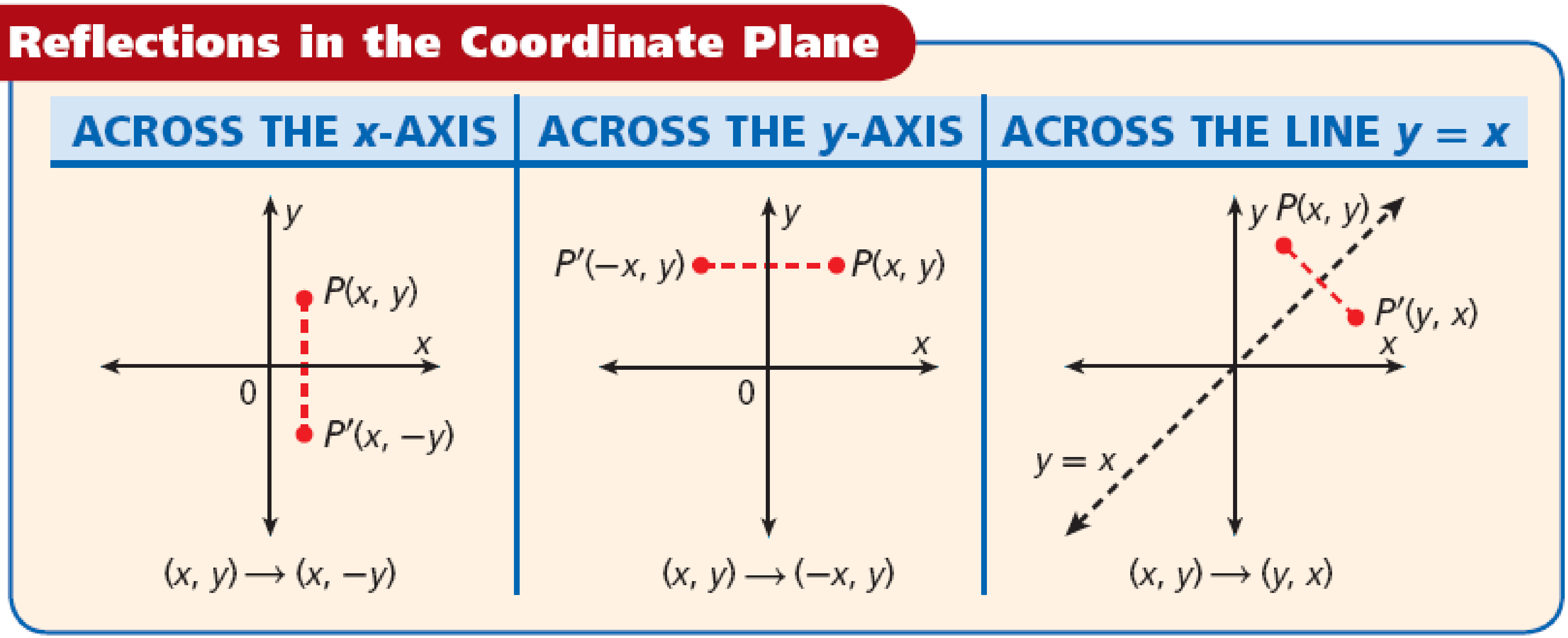
b.



translation; *MNOP* → *M′N′O′P′*

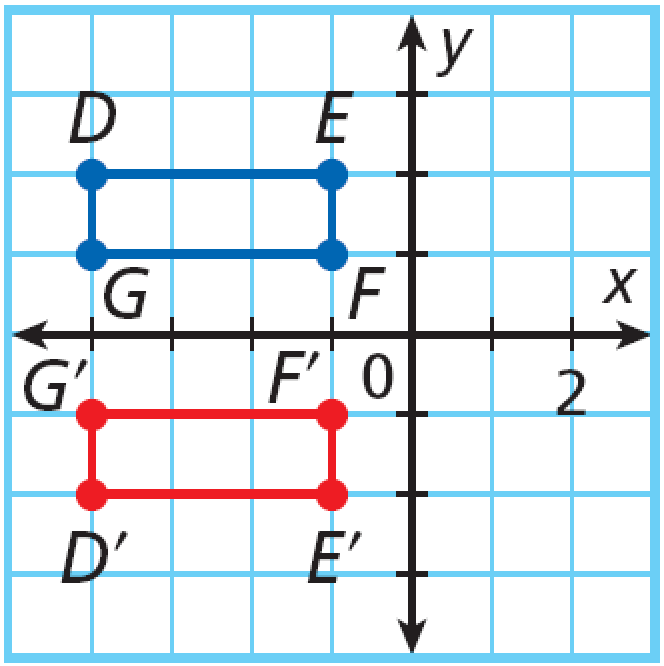
rotation; Δ*XYZ* → Δ*X′Y′Z′*

Reflection Rules



Example: 2

Identify the transformation.

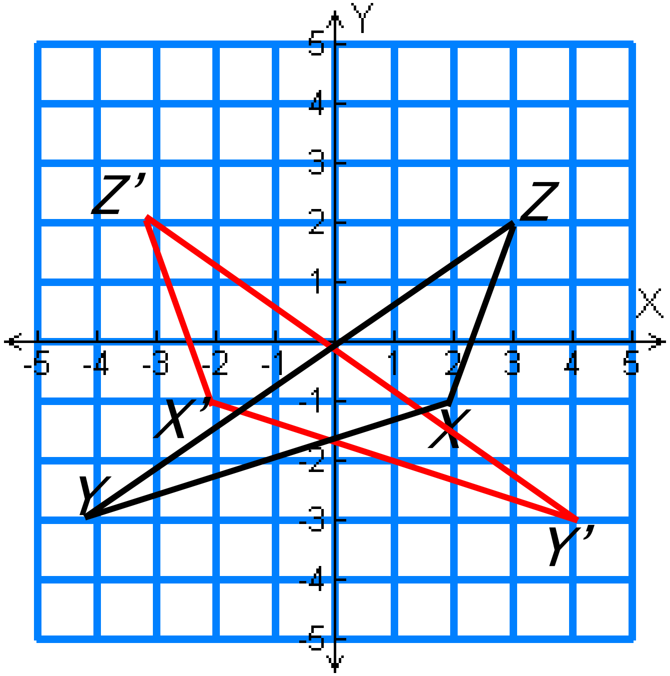


Reflection across the x axis,
DEFG $\rightarrow$ ***D′E′F′G′***

Example: 3

Reflect the figure with the given vertices across the given line.

X(2, –1), *Y*(–4, –3), *Z*(3, 2); *y*-axis



The reflection of (*x*, *y*) is (–*x*, *y*).

X(2,–1) → *X'*(–2, –1)

Y(–4,–3) → *Y'*(4, –3)

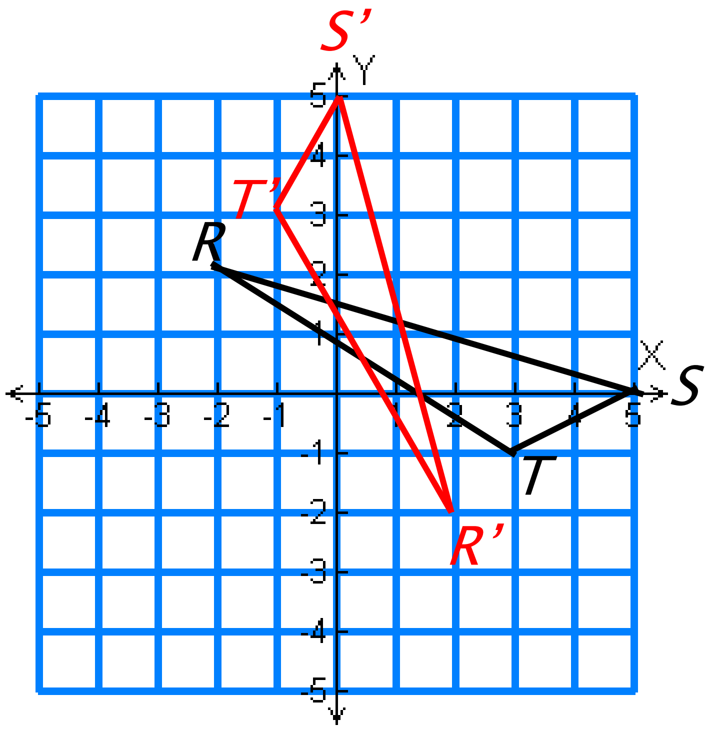
Z(3, 2) → *Z'*(–3, 2)

Graph the image and preimage.

Example: 4

Reflect the figure with the given vertices across the given line.

$R(-2, 2)$, $S(5, 0)$, $T(3, -1)$; $y = x$



The reflection of (x, y) is (y, x).

$R(-2, 2) \rightarrow R'(2, -2)$

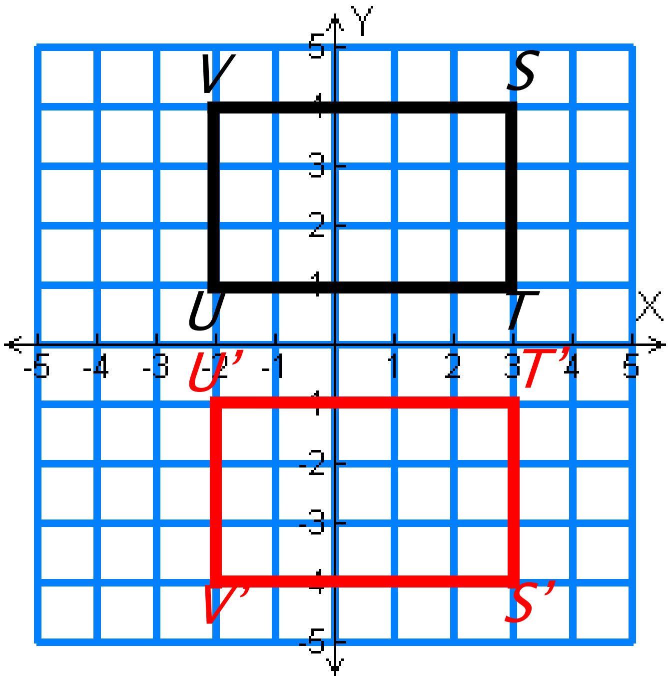
$S(5, 0) \rightarrow S'(0, 5)$

$T(3, -1) \rightarrow T'(-1, 3)$

Graph the image and preimage.

Example: 5

Reflect the rectangle with vertices *S*(3, 4), *T*(3, 1), *U*(–2, 1) and *V*(–2, 4) across the *x*–axis.



The reflection of (*x*, *y*) is (*x*, –*y*).

S(3, 4) → *S'*(3, –4)

T(3, 1) → *T'*(3, –1)

U(–2, 1) → *U'*(–2, –1)

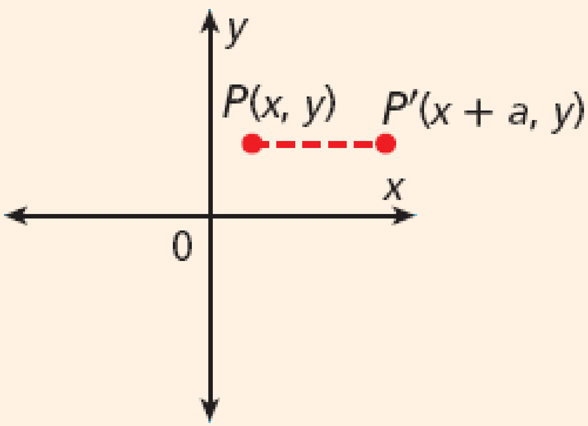
V(–2, 4) → *V'*(–2, –4)

Graph the image and preimage.

Translation Rules

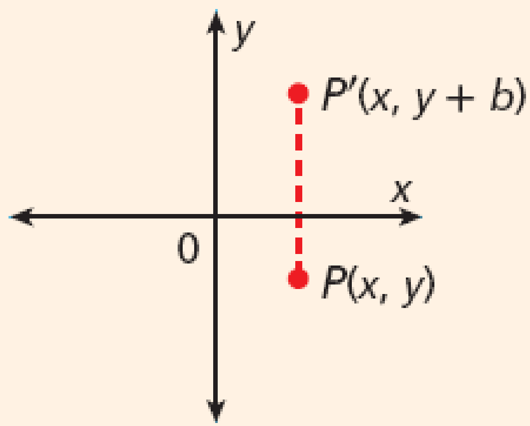
Translations in the Coordinate Plane

HORIZONTAL TRANSLATION ALONG VECTOR ⟨*a*, 0⟩



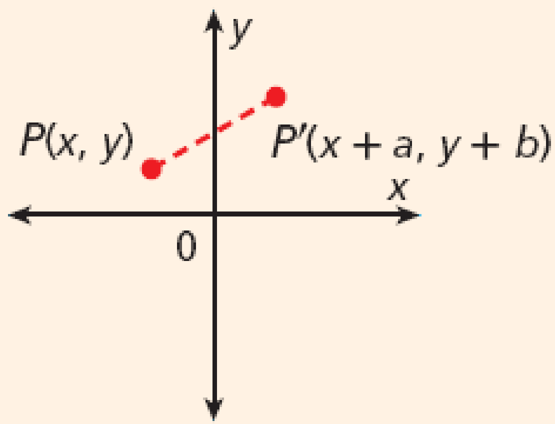
$(x, y) \longrightarrow (x + a, y)$

VERTICAL TRANSLATION ALONG VECTOR ⟨0, *b*⟩



$(x, y) \longrightarrow (x, y + b)$

GENERAL TRANSLATION ALONG VECTOR ⟨*a*, *b*⟩



$(x, y) \longrightarrow (x + a, y + b)$

To find coordinates for the image of a figure in a translation, add a to the x-coordinates of the preimage and add b to the y-coordinates of the preimage.

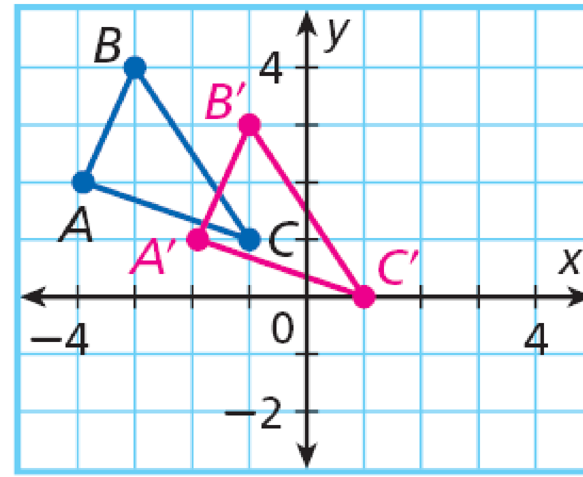
Translations can also be described by a rule such as $(x, y) \rightarrow (x + a, y + b)$.

Example: 6

Find the coordinates for the image of Δ*ABC* after the translation (*x*, *y*) → (*x* + 2, *y* - 1). Draw the pre image and image.

Step 1 Find the coordinates of Δ*ABC*.

The vertices of Δ*ABC* are *A*(–4, 2), *B*(–3, 4), *C*(–1, 1).



Step 2 Apply the rule to find the vertices of the image.

A′(–4 + 2, 2 – 1) = *A*′(–2, 1)

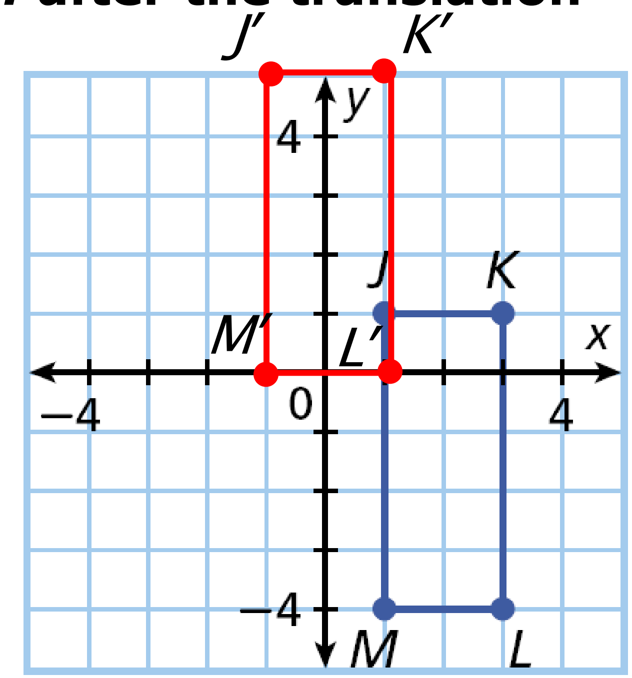
B′(–3 + 2, 4 – 1) = *B*′(–1, 3)

C′(–1 + 2, 1 – 1) = *C*′(1, 0)

Step 3 Plot the points. Then finish drawing the image by using a straightedge to connect the vertices.

Example: 7

Find the coordinates for the image of *JKLM* after the translation $(x, y) \rightarrow (x - 2, y + 4)$. Draw the image.



Step 1 Find the coordinates of *JKLM*.

The vertices of *JKLM* are *J*(1, 1), *K*(3, 1), *L*(3, –4), *M*(1, –4), .

Step 2 Apply the rule to find the vertices of the image.

$J'(1 - 2, 1 + 4) = J'(-1, 5)$
$K'(3 - 2, 1 + 4) = K'(1, 5)$
$L'(3 - 2, -4 + 4) = L'(1, 0)$
$M'(1 - 2, -4 + 4) = M'(-1, 0)$

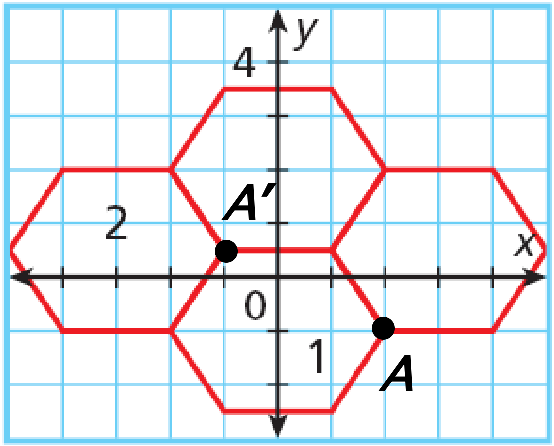
Step 3 Plot the points. Then finish drawing the image by using a straightedge to connect the vertices.

Example: 8

The figure shows part of a tile floor. Write a rule for the translation of hexagon 1 to hexagon 2.

Step 1 Choose two points.

Choose a Point *A* on the preimage and a corresponding Point *A′* on the image. *A* has coordinate (2, –1) and *A′* has coordinates $\left(-1, \frac{1}{2}\right)$.



Step 2 Translate.

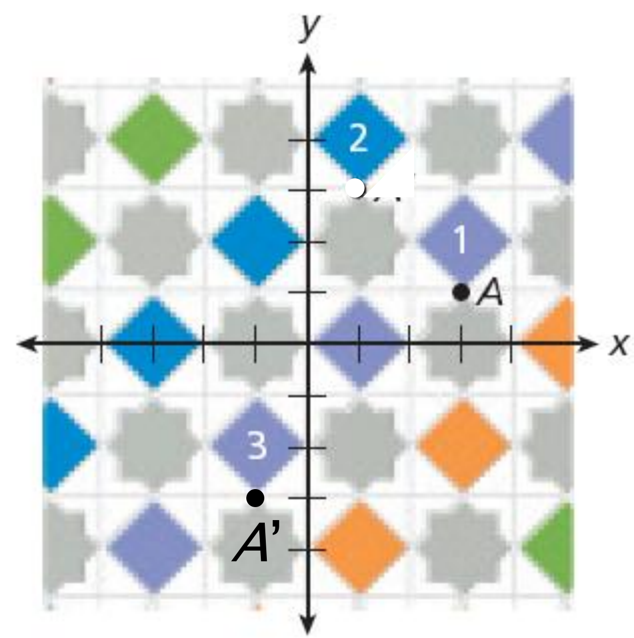
To translate *A* to *A′*, 2 units are subtracted from the *x*-coordinate and $1\frac{1}{2}$ units are added to the *y*-coordinate. Therefore, the translation rule is $(x, y) \rightarrow (x - 3, y + 1\frac{1}{2})$.

Example: 9

Use the diagram to write a rule for the translation of square 1 to square 3.

Step 1 Choose two points.

Choose a Point *A* on the preimage and a corresponding Point A′ on the image. *A* has coordinate (3, 1) and *A*′ has coordinates (–1, –3).

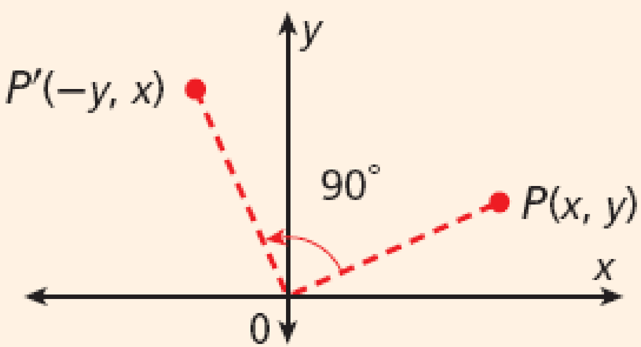


Step 2 Translate.
To translate *A* to *A*′, 4 units are subtracted from the *x*-coordinate and 4 units are subtracted from the *y*-coordinate. Therefore, the translation rule is $(x, y) \rightarrow (x - 4, y - 4)$.

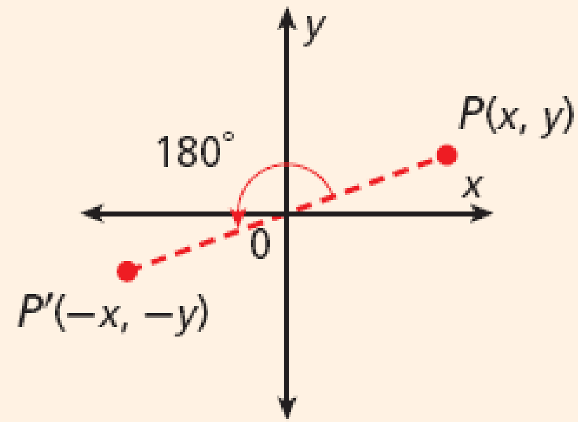
Rotation Rules

Rotations in the Coordinate Plane

BY 90° ABOUT THE ORIGIN	BY 180° ABOUT THE ORIGIN
Counterclockwise	



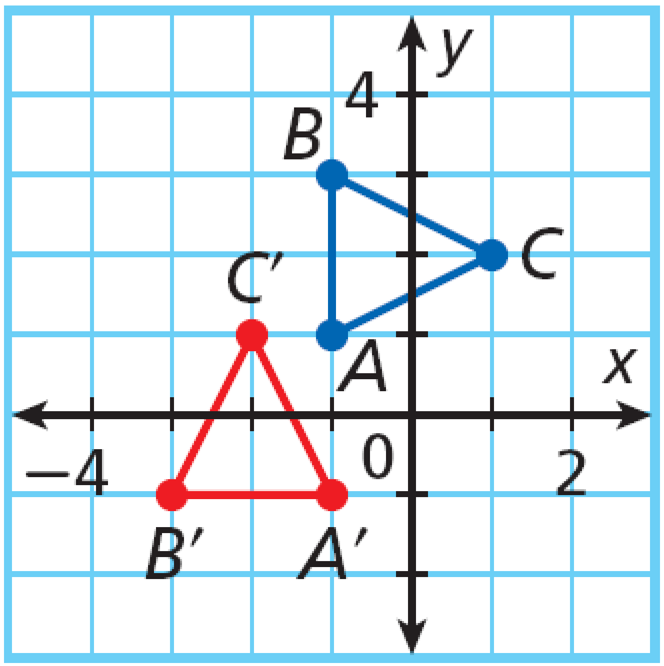
$(x, y) \longrightarrow (-y, x)$



$(x, y) \longrightarrow (-x, -y)$

Example: 10

Describe the transformation.



90° rotation counterclockwise,

$\Delta ABC \to \Delta A'B'C'$

Example: 11

Rotate ΔABC by 90° about the origin.

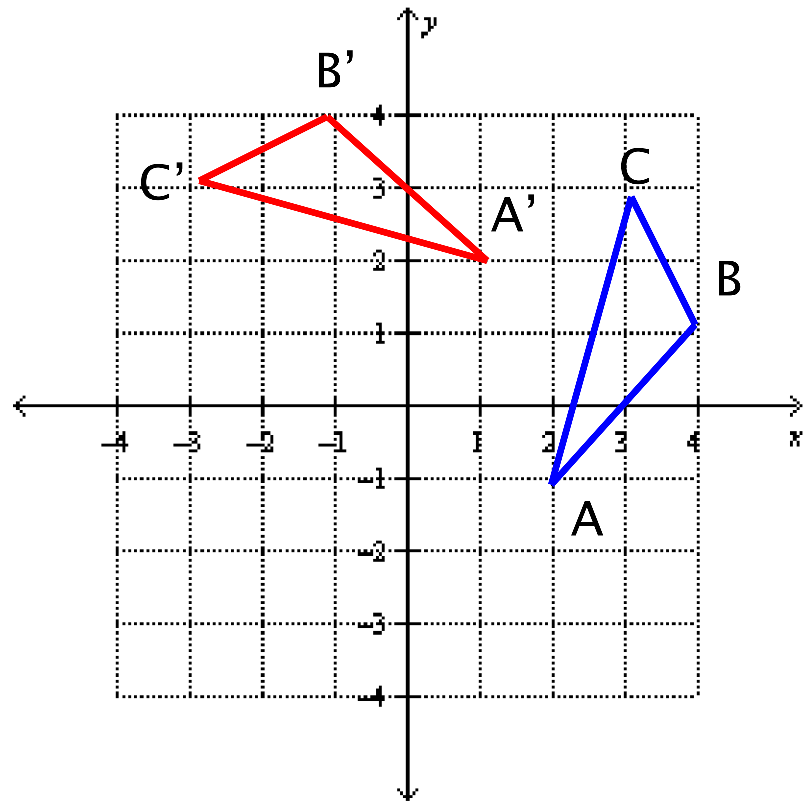
The rotation of (x, y) is $(-y, x)$.

$A(2, -1) \rightarrow A'(1, 2)$

$B(4, 1) \rightarrow B'(-1, 4)$

$C(3, 3) \rightarrow C'(-3, 3)$

Graph the preimage and image.



Example: 12

Rotate Δ*JKL* with vertices *J*(2, 2), *K*(4, –5), and *L*(–1, 6) by 180° about the origin.

The rotation of (*x*, *y*) is (–*x*, –*y*).

J(3, 5) ⟶ *J'*(–3, –5)

K(4, –5) ⟶ *K'*(–4, 5)

L(–1, 6) ⟶ *L'*(1, –6)

Graph the preimage and image.